# Enriques quotients of the universal cover of E[n] of an Enriques surface E



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## 1. Introduction

An Enriques surface $E$ is a compact complex surface with $H^1(E, \mathcal{O}_E) = 0$, $H^2(E, \mathcal{O}_E) = 0$, and $\omega_E^{\otimes 2} \simeq \mathcal{O}_E$. Let $E^{[n]}$ be the Hilbert scheme of $n$ points of $E$. We fix the universal cover $\pi : X \to E^{[n]}$ and its covering involution $\rho$. It is known that $\pi_1(E^{[n]}) = 2$ and $X$ is a Calabi-Yau manifold.

**Definition 1.1.** *A variety $Y$ is called an Enriques quotient of $X$ if there is an Enriques surface $E'$ and a free involution $\tau$ of $X$ such that $Y = X/\langle\tau\rangle \cong E'^{[n]}$. Here we will call two Enriques quotients of $X$ distinct if they are not isomorphic to each other.*

We count the number of distinct Enriques quotients of $X$.

## 2. Ohashi's result

When $n = 1$, Ohashi obtained the following theorem (see [1, Theorem 0.1]).

**Theorem 2.1.** *For any nonnegative integer $l$, there exists a $K3$ surface with exactly $2^{l+10}$ distinct Enriques quotients. In particular, there does not exist a universal bound for the number of distinct Enriques quotients of a $K3$ surface.*

## 3. Main Theorem 1

When $n \geqq 3$, the situation is totally different from Ohashi's result (see[2, Theorem 1.7]). We get the following Theorem.

**Theorem 3.1.** *If $\tau$ is a free involution of $X$ such that $X/\langle\tau\rangle$ is an Enriques quotient of $X$, then $\tau = \rho$. In particular the number of distinct Enriques quotients of $X$ is one.*

## 4. Stragety

i) We show that for $n \geq 3$, the covering involution of $\pi : X \to E^{[n]}$ acts on $H^2(X, \mathbb{C})$ as id and $H^{2n-1,1}(X, \mathbb{C})$ as $-$id. Remark $n = 2$, the covering involution of does not acts on $\mathrm{H}^2(X, \mathbb{C})$ as id.

ii) We show that for $n \geq 2$, if an automorphism $\varphi$ of $X$ acts on $H^2(X, \mathbb{C})$ as identity, then $\varphi$ is a lift of a natural automorphism of $E^{[n]}$.

By using the above, we get Main Theorem 1.

## 5. Main Theorem 2

When $n = 2$, we get the follwing Theorem.

**Theorem 5.1.**
i) For two Enriques surfaces $E$ and $E'$,
if $E^{[2]} \cong E'^{[2]}$, then $E \cong E'$.
ii) $\mathrm{Aut}(E^{[2]}) \cong \mathrm{Aut}(E)$, i.e. all automorphisms of $\mathrm{Aut}(E^{[2]})$ are the natural automorphisms.

**Remark 5.2.**
When $n = 2$, we did not yet count the number of distinct Enriques quotients of $X$.

## 6. References

[1] H. Ohashi: On the number of Enriques quotients of a K3 surface. Publ. Res. Inst. Math. Sci. 43 (2007), no. 1, 181-200. 14J28.
[2] T. Hayashi: Universal covering calabi-yau manifolds of the Hilbert schemes of n points of Enriques surfaces. arXiv:1502.02231.